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Momentum Dependence of the Penguin Interaction

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Abstract

We have considered the penguin interaction contribution to $K \to 2\pi$ decays. In particular, we have investigated the effect of the momentum dependence of the penguin coefficient. Our analysis is performed within the Chiral Quark Model where quarks are coupled to the pseudoscalar mesons, which means that hadronic matrix elements can be calculated in terms of quark loop diagrams.

We have inserted the momentum dependent penguin coefficient into the relevant quark loop diagrams for $K \to \pi$. We discuss two possible prescriptions for performing the calculations, and conclude that the momentum dependence of the penguin coefficient increases the amplitude by 10-20 %. In any case, the (CP-conserving) penguin contribution is very sensitive to the values chosen for the involved parameters.

1. Introduction

The penguin (gluonic monopole) diagram was proposed as an explanation of the $\Delta I = 1/2$ rule [1]. This diagram induced pure $\Delta I = 1/2$ four quark operators into the effective Lagrangian relevant for $K \to 2\pi$ decays and other $\Delta S = 1$ transitions. In general, non-leptonic decays are described by an effective Lagrangian[2]

$$\mathcal{L}(\Delta S = 1) = -\frac{1}{\sqrt{2}} G_F \lambda_u \sum_i C_i Q_i \quad , \tag{1}$$

where G_F is Fermis coupling constant and $\lambda_u = V_{su} V_{du}^*$ is the relevant Kobayashi-Maskawa[3] factor for CP-conserving $\Delta S = 1$ transitions. In such effective Lagrangians, the heavy mass scales are integrated out and their effects are contained in the Wilson coefficients C_i. The quark operators Q_i involve the three light quarks $q = u, d, s$. The coefficients will in general include short distance QCD effects calculated by means of perturbation theory and the renormalization group equations (RGE). Both the coefficients C_i and the matrix elements of the quark operators depend on the renormalization scale μ (taken to be of order 1 GeV), in such a way that the physical processes are independent of μ. Concerning the discussion on the importance of the penguin interaction for the $\Delta I = 1/2$ rule within the standard approach, we refer to the literature[2, 4, 5, 6, 7, 8, 9, 10]. In this paper, we will revisit this issue from some more unconventional point of view, and ask if some effects have been left out.

To calculate the matrix elements of quark operators between physical hadronic states is in general a difficult task, and one normally uses various models or assumptions. In this paper we will use the Chiral Quark Model (χQM), advocated by many authors [11, 12, 13]. The model consists of the ordinary QCD Lagrangian and in addition a term $\mathcal{L}_\chi$. This new term includes the Goldstone meson octet in a chiral invariant way, and provides meson-quark couplings that makes it possible to calculate matrix elements of quark operators in terms of quark loop diagrams. The model is thought to be applicable for quark momenta below some scale of the order Λ_χ, the chiral symmetry breaking scale. Although this is a model, we find it very interesting because it reproduces the chiral Lagrangian terms for strong interactions to good accuracy to order p^4 [12]. As a field-theoretical model, it can be formulated in terms of path integrals. Integrating out the quarks, one obtains chiral Lagrangians. Alternatively, integrating out the Goldstone mesons, one obtains Nambu type models[14, 15].

In order to predict physical amplitudes like $K \to 2\pi$, one has to match the high energy description, contained in the coefficients C_i, with the low

energy description. This matching should be done at some scale μ where both descriptions are valid. How to do this has been a delicate question, and especially for the penguin contributions because the coefficients are very sensitive to the quantity m_c/μ, where m_c is the charm-quark mass.

In this paper we will focus on the following aspect of the effective Lagrangian (1): The coefficients C_i in (1) originates from some Feynman diagrams and will in principle depend on the external quark momenta, that is, the quark momenta corresponding to the quark fields in Q_i in (1). Working within the χQM, we might keep the full momentum dependence of the penguin coefficient, calculated to lowest order in the strong coupling constant. Being a field theoretic model, the χQM can of course account for such momentum dependent effects. Unfortunately, our calculation will have some limitations. Especially, we lose the RGE analysis of the Wilson coefficient because the RGE analysis does not keep track of the individual quark momenta, only their overall scale. Thus, our procedure will be less systematic than the standard one [2, 4, 5, 6, 13]. Still, because the standard method might have some difficulties with the matching around 1 GeV, we find it fruitful to perform such a calculation to see if there are effects which are lost in the standard approach.

2. The penguin interaction at quark level

The penguin diagram is shown in Fig.1, and induces an effective interaction (in the CP-conserving case)

$$\mathcal{L}_P \;=\; -\sqrt{2}\, G_F\, \lambda_u\, C_P\, Q_P \tag{2}$$

where C_P is the Wilson coefficient of the operator

$$Q_P \;=\; (\bar{d}\gamma_\nu L t^a s)\, \bar{q}\gamma^\nu t^a q \, , \tag{3}$$

where a sum over $q = u, d, s$ is understood. The t^a's are the colour matrices (for $a = 1, .., 8$), and g_s is the strong coupling. For the CP-conserving case considered here, the dominating part is due to u- and c-quarks running in the penguin loop, and the following expression is obtained in the leading logarithmic approximation before any RGE analysis is performed:

$$C_P \;=\; -\frac{\alpha_s}{3\pi}\, \ln\frac{m_c^2}{\mu^2} \tag{4}$$

where m_c is the c-quark mass. Going beyond the leading logarithmic approximation, one finds the following expression [16, 10]:

$$C_P \;=\; -\frac{\alpha_s}{3\pi}\, (\ln\frac{m_c^2}{\mu^2} + \frac{5}{3}) \, . \tag{5}$$

where $\alpha_s = g_s^2/4\pi$. Here the constant next to leading term is numerically as important as the leading logarithm. This illustrates the difficulties in treating the penguin diagram perturbatively.

The expression (5) still does not take into account the variation of C_P with the gluon momentum p. This variation can be expressed as [17]:

$$C_P(p^2) = (-\frac{\alpha_s}{3\pi})\, 6 \int_0^1 dt\, t(1-t) \int_{m_u^2}^{m_c^2} \frac{d\rho}{\rho - t(1-t)p^2} \;, \qquad (6)$$

which exhibits the GIM mechanism [18] explicitly. The variation of this function with momentum is shown in Fig.2. Eq. (5) is obtained from (6) if one uses the identification $p^2 = -\mu^2$, and the approximation $m_c^2 \gg \mu^2 \gg m_u^2$. For momenta of order M_W, the coefficient C_P is more complicated[10], but this is irrelevant for us here. Using dimensional regularization $C_P(p^2)$ takes the following form:

$$C_P(p^2) = (-\frac{\alpha_s}{3\pi})\, 6\, \Gamma(\epsilon)\, \tau(\epsilon) \int_0^1 dt\, t(1-t) \left[(-D_u)^{-\epsilon} - (-D_c)^{-\epsilon}\right] , \qquad (7)$$

where Γ is the gamma-function satisfying $\Gamma(n) = (n-1)!$ for an integer n. For space dimension $D = 4 - 2\epsilon$, one has $\Gamma(\epsilon) = 1/\epsilon + constant$. Moreover, $\tau(\epsilon) = (-4\pi\mu^2)^\epsilon$, where μ is a parameter with dimension mass which enters the calculation within the dimensional regularization scheme. This parameter may be identified with the renormalization scale. The quark masses enters the expression (7) through the quantity

$$D_q = m_q^2 - t(1-t)p^2 \;, \qquad (8)$$

for $q = u, c$.

Using the SU(3) matrix relation

$$t^a_{ij} t^a_{kl} \;=\; \frac{1}{2}(\delta_{il}\delta_{jk} \;-\; \frac{1}{N_c}\delta_{ij}\delta_{kl}) \;, \qquad (9)$$

splitting the quark current in (3) in a left- and right-handed part and using a Fierz transformation, one obtains four different contributions to $\mathcal{L}(\Delta S = 1)$. The most important penguin four quark operator is Q_6, which in the Fierz-transformed version is a product of two quark densities, involving both left- and right-handed quark fields:

$$Q_6 \;=\; -8 \sum_q (\bar{d}_L q_R)\, (\bar{q}_R s_L) \;, \qquad (10)$$

where the sum runs over $q = u, d, s$. In the naive leading logarithmic approximation (without RGE), we can do the following identification.

$$C_6 \;=\; \frac{1}{4}\, C_P \qquad (11)$$

The expression for C_6 including RGE can be found in the literature [2, 5, 13, 19]. There are also penguin operators which are products of two left-handed currents, like

$$Q_4 \;=\; 4\sum_q (\bar{d}_L\gamma^\mu q_L)\,(\bar{q}_L\gamma_\mu s_L)\;, \tag{12}$$

but these are not so important numerically.

In addition to penguin operators, there is an important pure $\Delta I = 1/2$ operator of left-left type which is $Q_- = Q_2 - Q_1$, where

$$Q_1 \;=\; 4\,(\bar{d}_L\gamma^\mu s_L)\,(\bar{u}_L\gamma_\mu u_L) \;\text{ and }\; Q_2 \;=\; 4\,(\bar{d}_L\gamma^\mu u_L)\,(\bar{u}_L\gamma_\mu s_L) \tag{13}$$

are quark operators contained in (1).

3. The Chiral Quark Model

In the Chiral Quark Model (χQM) [11, 12, 13], chiral-symmetry breaking is thought to be taken into account by adding an extra term $\mathcal{L}_\chi$ to ordinary QCD:

$$\mathcal{L}_{QCD} \to \mathcal{L}_{QCD\chi} = \mathcal{L}_{QCD} + \mathcal{L}_\chi \quad ; \quad \mathcal{L}_{QCD} = \mathcal{L}_f + \mathcal{L}_G \;, \tag{14}$$

where

$$\mathcal{L}_f = \bar{q}(i\gamma\cdot D - \mathcal{M}_q)q \quad ; \qquad q \;=\; \left(\begin{array}{c} u \\ d \\ s \end{array}\right) \;, \tag{15}$$

$\mathcal{M}_q$ is the *current* quark mass-matrix, $\mathcal{L}_G$ is the pure gluonic part of QCD, and

$$\mathcal{L}_\chi = -M\,(\overline{q_L}\Sigma q_R + \overline{q_R}\Sigma^\dagger q_L)\;. \tag{16}$$

The constant M in (16) is interpreted as the *constituent* quark mass, thought to be of order 200 - 300 MeV. Note that the constituent and current masses are connected to different terms in the Lagrangian, with different transformation properties. The quantity Σ contains the Goldstone-octet fields π^a:

$$\Sigma = exp(i\sum_a \lambda^a\pi^a/f) \;\;, \tag{17}$$

where λ^a are the Gell-Mann matrices, and $f = f_\pi = 93$ MeV is the pion decay constant. The term $\mathcal{L}_\chi$ introduces meson-quark couplings. This means that the quarks can be integrated out and the coefficients of the various terms in the effective meson theory, the chiral Lagrangian, are calculable from $\mathcal{L}_{QCD\chi}$ [12]. It has also been found to be suitable for calculating hadronic matrix elements of operators obtained from the weak sector[13, 20], like in eq. (1).

There are several versions of the χQM in the literature. Note that in eqs. (14)-(17), there is no kinetic term for the mesons. Thus, the meson fields are external. They propagate only after the quarks are integrated out. The χQM is thought to apply for momenta of the order and below the scale of chiral-symmetry breaking, which we define to be

$$\Lambda_\chi = 2\pi f_\pi \sqrt{6/N_c} \, , \tag{18}$$

where $N_c = 3$ is the number of colours (-numerically $\Lambda_\chi = 0.83$ GeV). Therefore a physical ultraviolet cut-off Λ of the order Λ_χ is sometimes used within the χQM to parametrize loop integrals which would otherwise be divergent. Such divergences will be buried in the physical f_π and quark condensate (as shown below), and are not removed by counterterms as in ordinary field theories. One should note that Λ and Λ_χ are in principle different quantities. One may also use dimensional regularization within the χQM, as shown in [20]. Even if dimensional regularization is used, Λ_χ is still some kind of effective cut-off scale of the χQM, just as for chiral perturbation theory. (Analogously, the W-mass is some effective cut-off scale of electroweak interactions, even if dimensional regularization is used).

There are a priori several ways to introduce an ultraviolet cut-off Λ within the χQM. The simplest way is to use an explicit (sharp) cut-off in the integration over virtual Euclidean momenta. However, this prescription violates translation invariance, and will only give a unique result for the leading term. One might try a Pauli-Villars type of cut-off[21], which turns out to be technically cumbersome in our case. A good alternative is proper time regularization, which has already been used [22, 15] in χQM calculations. In this case one uses the following replacement for the propagator denominator (for Euclidean momenta):

$$\frac{1}{p^2 + M^2} \; \rightarrow \; \int_\xi^\infty d\tau \, exp[-\tau(p^2 + M^2)] \, , \tag{19}$$

where $\xi = 1/\Lambda^2$.

The cut-off Λ can not be chosen freely. Indeed the χQM[13, 15] provides a relation between Λ, M and f_π, and eventually gluon condensates. Such a relation is obtained because f_π, entering the meson- quark coupling $\sim M\gamma_5/f_\pi$, is also given by a quark loop diagram for $\pi \rightarrow W$(virtual), and one obtains:

$$f_\pi^{(0)} \; = \; \frac{N_c M^2}{4\pi^2 f} [\, \hat{f}_\pi \; + \; \frac{\pi^2}{6 N_c M^4} < \frac{\alpha_s}{\pi} G^2 > + \cdots] \, , \tag{20}$$

where $< \frac{\alpha_s}{\pi} G^2 >$ is the two gluon condensate, and the dots indicate higher gluon condensates. The value of Λ will depend on how many gluon condensates which are kept in (20). In the end both f and $f_\pi^{(0)}$ will, in the limit

$m_{u,d} \rightarrow 0$, be identified by f_π, but at intermediate stages one might need to distinguish them for technical reasons (-we also have $f_K = f_\pi^{(0)}$ in the limit $m_{u,d,s} \rightarrow 0$).

The dimensionless quantity $\hat{f}_\pi$ has the leading behaviour $\sim \ln(\frac{\Lambda^2}{M^2})$ in a cut-off type regularization. Its explicit form will depend on the way the cut-off is introduced. Within dimensional regularization, $\hat{f}_\pi = \Gamma(\epsilon)(4\pi\tilde{\mu}^2/M^2)^\epsilon$, where $\tilde{\mu}$ is a parameter with dimension mass, occurring owing to the use of dimensional regularization in the χQM. Thus, the would be divergent (for $\Lambda \rightarrow \infty$) logarithmic term (or $1/\epsilon$ term) is absorbed in the physical pion decay constant f_π [13, 20].

The quark condensate is within χQM given by[13, 15, 21, 23]

$$< \bar{q}q > = -(\tilde{\mu}^2)^\epsilon \int \frac{d^D p}{(2\pi)^D} Tr[iS(p)] \; = \; \frac{N_c M}{4\pi^2} C_q - \frac{1}{12M} < \frac{\alpha_s}{\pi} G^2 > + ... \; , \tag{21}$$

where $S(p)$ is the quark propagator in an external gluon field[23]. The quantity C_q depends on the regularization prescription. For a cut-off type regularization, $C_q = -\Lambda^2 + M^2 \ln \frac{\Lambda^2}{M^2} + \cdots$ Within dimensional regularization, $C_q = -M^2 \Gamma(-1+\epsilon)(4\pi\tilde{\mu}^2/M^2)^\epsilon$. Equation (21) puts (in addition to eq.(20)) further restrictions on the parameters of the χQM.

4. The standard $K \rightarrow \pi$ amplitude.

The $K \rightarrow 2\pi$ amplitudes can in general be described in terms of chiral Lagrangians. Knowing the structure of chiral Lagrangians, the octet part of the $K \rightarrow 2\pi$ amplitude can be found from the virtual $K \rightarrow \pi$ amplitude [4, 5, 6, 13]. Following [13] we define the coupling constant $g_8^{(1/2)}$ of the octet chiral Lagrangian. In term of this, the virtual $K \rightarrow \pi$ amplitude can be written:

$$\mathcal{M}(K^- \rightarrow \pi^-)_8 \; = \; -\sqrt{2} G_F \lambda_u \, k^2 \, g_8^{(1/2)} \, f_K \, f_\pi \; , \tag{22}$$

where k is the momentum of the virtual meson transition.

The standard result for the penguin contribution, mainly due to Q_6, may be written as [13]

$$g(Q_6)_S \equiv g_8^{(1/2)}(Q_6)_S = -16 \, ReC_6(\mu) \, (\frac{\langle \overline{q}q \rangle}{f^3})^2 \, L_5 \; , \tag{23}$$

where L_5 is the coupling constant of the relevant term in the strong chiral Lagrangian of $\mathcal{O}(p^4)$. Within the χQM one obtains [15, 20]

$$L_5 \; = \; - \frac{f^3 f_\pi^{(0)}}{8M \langle \overline{q}q \rangle} \, [1 - \rho] \; . \tag{24}$$

where the quantity ρ is given (up to $\mathcal{O}(M^4/\Lambda_\chi^4)$) by

$$\rho \;=\; 6\frac{M^2}{\Lambda_\chi^2}\frac{f}{f_\pi^{(0)}}\;; \qquad \text{or} \qquad \rho \;=\; \frac{1}{\Gamma(0,x)}\;, \tag{25}$$

where the first expression is used within dimensional regularization in [20], and the last one involving the incomplete Γ-function, $\Gamma(0,x)$ with $x = M^2/\Lambda^2$, is used within proper time regularization in [15]. In fact the two expressions for ρ are equivalent when using eq. (18), and (20) with $\hat{f}_\pi = \Gamma(0,x)$. Keeping $f_\pi^{(0)}/f = 1$ in the first expression for ρ in (25), the expression for L_5 in (24) is very sensitive to M.

Within the χQM, the quantity $g_8^{(1/2)}(Q_6)$ is given by the diagrams in Fig.3. The $Ks\bar{u}$ and $\pi\bar{u}d$ vertices are obtained from $\mathcal{L}_\chi$ in (16)-(17). These diagrams contain terms independent of the meson momentum. However, such terms cancel in accordance with chiral symmetry [4] . The leading physical amplitude corresponds to the sum of the terms of order k^2. The leading term in the expression for L_5 in (24) corresponds to diagram 3a, while the formally non-leading term $\sim \rho$ in (24) corresponds to diagram 3b. Numerically, however, there is a rather strong cancellation between these two terms for values of M bigger than 250 MeV. We observe that values of $M \simeq$ 330 MeV or bigger are not acceptable (while keeping $f_\pi^{(0)}/f = 1$) because they correspond to zero or even slightly negative values of L_5 in (24). Using explicitly the second expression for ρ in (25) would lead to a reduced value of L_5 for M smaller than 250 MeV. One should however take into account that $\Gamma(0,x) = \hat{f}_\pi$ is related to the physical f_π through (20). It is a priori an open question if the strong cancellation between the leading diagram 3a and the nonleading 3b persists when the momentum variation of the penguin coefficient is taken into account (- diagram 3c serves to cancel the momentum independent parts of 3a and 3b).

In [13] the expression (24) was not used. Instead the value $L_5 = 1.8\times 10^{-3}$ was extracted from other arguments. Moreover, using $\mu \simeq M$, and the scale independent quark condensate $< \bar{q}q >= (-194\,MeV)^3$, it is found that $g(Q_6)_S = 0.26$. In general, one obtains bigger values for the quark condensate if one uses the expression in [5],

$$\langle \overline{q}q\rangle(\mu) \;=\; -\,\frac{f_\pi^2 m_\pi^2}{m_u(\mu)+m_d(\mu)}\;\;. \tag{26}$$

Alternatively, using the values[24] $< \overline{q}q >= (-235 MeV)^3$, $L_5 = 1.4\times 10^{-3}$ at the scale 1 GeV (- corresponding to $M \simeq 245$ MeV if (24) is used), and using [19] $C_6(\mu = 1 GeV) = -0.026$ for Λ_{QCD}= 400 MeV, one obtains $g(Q_6)_S = 0.15$. However, bigger values might be obtained. For $\mu \simeq 0.8$ GeV and $\Lambda_{QCD} = 350$ MeV, $C_6 \simeq -0.1$, and using in addition [20, 25] $M \simeq 200$

MeV, one obtains $g(Q_6)_S \simeq 0.8$. (Note that, from (5) and (11) one obtains $C_6 \simeq -0.05$ for $\mu \simeq \Lambda_\chi$)

The biggest contribution to $g_8^{(1/2)}$ is coming from the four quark operator Q_- of left-left type (-see (13)), which gives the amplitude[13]

$$g_8^{(1/2)}(Q_-) \; = \; \frac{1}{2} C_- [1 - \frac{1}{N_c}(1-\delta)] \tag{27}$$

where C_- is the Wilson coefficient of the operator Q_-, which is $\simeq 2$ at $\mu \simeq$ 0.8 GeV, and the quantity δ represents non-factorizable gluon condensate corrections:

$$\delta \equiv \frac{N_c < \frac{\alpha_s}{\pi} G^2 >}{32\pi^2 f_\pi^4} \; . \tag{28}$$

Writing $< \frac{\alpha_s}{\pi} G^2 >= \eta^4$, the quantity η is of the order 400 MeV, which gives a value of δ around 3 ($\delta \simeq 2.6\,, 3.0$, and 3.7, for $\eta = 376\,, 390$, and 410 MeV, respectively.) Using $\eta = 376$ MeV and $\mu \simeq 0.8$ GeV one obtains $g_8^{(1/2)}(Q_-) \simeq 1.6$, while for $\eta = 390$ MeV and $\mu \simeq 320$ MeV one obtains[13] $g_8^{(1/2)}(Q_-) \simeq 2.6$. Anyway, the prediction will be below the experimental value for the total $\Delta I = 1/2$ amplitude [5, 13]

$$g^{(1/2)}(Tot.)_{Exp.} \; = \; 5.1 \;\; . \tag{29}$$

5. Momentum dependent penguin coefficient

Now we will present the calculation using a momentum dependent penguin coefficient. In the chiral limit $m_{s,d} \to 0$, we obtain the following amplitude corresponding to the operator Q_6 from the diagram in Fig.3a,

$$\mathcal{M}_a(K^- \to \pi^-; Q_6) \; = \; \sqrt{2} G_F \lambda_u \, (-8) \, (\frac{N_c M}{f_\pi} \, \tilde{\mu}^{2\epsilon})^2 \; I_\chi \; , \tag{30}$$

where I_χ is the two loop integral

$$I_\chi \; = \; \int \frac{d^D p}{(2\pi)^D} \int \frac{d^D r}{(2\pi)^D} \, F(s', s) \, [C_P(p^2)] \, F(r', r) \; . \tag{31}$$

Here s and r are loop momenta and $k = r - r' = s - s'$ is the momentum of the virtual $K \to \pi$ transition, and $p = s - r$ is the momentum of the penguin gluon. The function F is given by

$$F(r', r) \; = \; \frac{r \cdot r' - M^2}{(r^2 - M^2)(r'^2 - M^2)} \tag{32}$$

where the trace in Dirac space has been taken within naive dimensional regularization, for simplicity. If C_P is taken as a constant, I_χ splits in the product of two one loop integrals, and we recover the result of other authors[2, 4, 13, 20]. Taking C_P as momentum dependent, I_χ is a two loop integral and cut-off procedures are in general cumbersome to implement. We have, however, found that if C_P is given as in (7), the integral can be calculated analytically within dimensional regularization (in $D = 4 - 2\epsilon$ dimensions). Then the contribution from the diagram in Fig.3a given by (30),(31) and (32) turns out to dominate over the contribution from the diagram in Fig.3b.

We have calculated the integral using Feynman parametrization in the usual way. The r integration is easily performed, while the remaining integration over p is the tricky part. Some details will be given in the Appendix. Within the approximation $m_c^2 \gg M^2$, we find the result:

$$I_\chi = (-\frac{\alpha_s}{3\pi})(\frac{i}{16\pi^2})^2 \, k^2 \, (-2) \, (\frac{1}{\epsilon})^2 \, m_c^2 \, [1 + \mathcal{O}(\frac{M^2}{m_c^2}) + \mathcal{O}(\epsilon)] \, , \tag{33}$$

Here we have dropped an irrelevant term constant with respect to the virtual meson momentum k. This constant cancels with the constant term obtained from the diagram in Fig.3b and 3c. We find that the $\sim k^2$ contribution from diagram 3b is $\sim M^2$ instead of $\sim m_c^2$. Examining the integral I_χ to order k^2, one observes (-see Appendix) that one factor $1/\epsilon$ comes from a quadratic divergence and will be related to the quark condensate, and the other $1/\epsilon$ comes from a logarithmic divergence and will be related to f_π. Now we can combine the result (33) with the equations (20) and (21), and we obtain a result for $\mathcal{M}_a(K^- \to \pi^-; Q_6)$ in (30). Then we can convert this result into the following contribution to $g_8^{(1/2)}$:

$$g(Q_6)_{DR} = -\frac{\alpha_s}{3\pi} \frac{\langle \overline{q}q \rangle m_c^2}{M^3 f_\pi^2} [1 + \mathcal{O}(\frac{M^2}{\Lambda_\chi^2}) + \mathcal{O}(\frac{M^2}{m_c^2})] \, , \tag{34}$$

where the $\mathcal{O}(M^2/\Lambda_\chi^2)$ term are coming from $\mathcal{O}(\epsilon)$ in (33). The result in (34) has to be compared to the standard amplitude

$$g(Q_6)_S = -\frac{\alpha_s}{3\pi} \, (\ln \frac{m_c^2}{\Lambda_\chi^2} + \frac{5}{3}) \, \frac{\langle \overline{q}q \rangle}{2M f_\pi^2} \, \left[1 - 6\frac{M^2}{\Lambda_\chi^2} \right] \, , \tag{35}$$

obtained from combining (5),(23) and (24). It might be surprising that the amplitude (34) is found to be proportional to m_c^2, which means a huge enhancement compared to (35) from the leading term alone. This behaviour reflects that loop momenta of order m_c are important in the last loop integral over p. The result is very sensitive to the involved parameters. Using $\alpha_s = \alpha_s(m_c^2) \simeq 0.38$, $\langle \overline{q}q \rangle = (-257 MeV)^3$ at $\mu = m_c$, we obtain $g_8^{(1/2)}(Q_6) \simeq 10$ for M= 0.25 GeV and $g(Q_6) \simeq 19$ for M= 0.2 GeV, which

is far above the experimental value for $g^{(1/2)}$! However, we observe from our calculations that the formally non-leading terms $\mathcal{O}(M^2/\Lambda_\chi^2)$ in (34), being for instance of the form

$$\frac{M^2}{\Lambda_\chi^2}\ln\frac{m_c^2}{\Lambda_\chi^2}\ , \qquad \frac{M^3 f_\pi^2}{\Lambda_\chi^2 < \bar{q}q >}\ln\frac{m_c^2}{\Lambda_\chi^2}\ , \qquad \text{and} \qquad \frac{M^2}{\Lambda_\chi^2}\ln\frac{\Lambda_\chi^2}{M^2} \tag{36}$$

might have sizeable coefficients of order 10. Obviously, big cancellations between leading and formally nonleading terms in (34) has to occur in order to get a realistic result. For momentum independent Wilson coefficients, dimensional regularization works pretty well [20]. Unfortunately, the coefficients (-and their signs!) of the formally nonleading terms in (36) have some ambiguity because dimensional regularization does not distinguish clearly between logarithmic divergences (associated with f_π) and quadratic divergences (associated with the quark condensate). Because of this, the result (34) is out of control and cannot be used for predicting $g(Q_6)$. As a curiosity, we observe that while the standard amplitude $g(Q_6)_S$ has a leading term proportional to $\Gamma(-1+\epsilon)\cdot\Gamma(\epsilon)$, the leading term in $g(Q_6)_{DR}$, being the result of a two loop integration, is proportional to $\Gamma(-1+3\epsilon)\cdot\Gamma(-\epsilon)\cdot F(\epsilon)$, where $F(\epsilon)$ is finite in the limit $\epsilon \to 0$.

The quadratic divergence is only appearing in left-right operators like Q_6 in (10). From penguin operators containing two left-handed currents, one obtains a product of two logarithmic divergences, and there is no ambiguity as for quadratic divergences. The standard contribution from Q_4 is (compare with (35))

$$g(Q_4)_S = \frac{\alpha_s}{12\pi}\,(\ln\frac{m_c^2}{\Lambda_\chi^2} + \frac{5}{3})\ . \tag{37}$$

Using the same method as for the Q_6, we obtain for a variable penguin coefficient within dimensional regularization (Only Fig.3a contributes in this case, but with axial currents acting at the weak vertices):

$$g(Q_4)_{DR} = \frac{\alpha_s}{12\pi}\left(\ln\frac{m_c^2}{M^2} + \frac{2}{3} - 9\frac{M^2}{\Lambda_\chi^2}\left[\ln\frac{m_c^2}{M^2}\,\ln\frac{m_c^2}{\Lambda_\chi^2} + \Delta\right]\right)\ , \tag{38}$$

where Δ parametrizes some (calculable) non-leading terms, and we have put $\mu = \tilde{\mu} = \Lambda_\chi$ in (7). Using the values $m_c = 1.4$ GeV, $M \simeq 200$ MeV and $\Lambda_\chi \simeq 830$ MeV, we obtain an increase by almost 70% of the leading term with respect to (37). However, this is partially compensated by the non-leading term $\sim M^2/\Lambda_\chi^2$, and the overall result is an increase of order 20 % of $g(Q_4)_{DR}$ with respect to $g(Q_4)_S$. Even if $g(Q_4)$ is small ($\simeq 0.05$), (38) gives an idea of how the penguin contributions behave when the variation of the penguin coefficients are taken into account. However, $g(Q_4)_{DR}$ is rather sensitive to variations in M.

Λ	M	$\alpha_s(\Lambda)$	f_π	$\langle\bar{q}q\rangle^{1/3}$	$g(Q_6)^0_P$	$g(Q_6)_P$	$g(Q_6)^M_P$
700	330	1.00	101	-189	-0.008	-0.006	- 0.01
830	330	0.70	126	-222	0.03	0.04	0.03
1000	330	0.53	155	-261	0.10	0.11	0.05
1200	330	0.44	185	-303	0.19	0.22	0.06
1400	330	0.38	211	-342	0.30	0.36	0.07
700	250	1.00	82	-185	0.04	0.05	0.09
830	250	0.70	98	-213	0.08	0.09	0.11
1000	250	0.53	115	-248	0.15	0.16	0.12
1200	250	0.44	133	-285	0.23	0.26	0.13
1400	250	0.38	148	-320	0.30	0.37	0.12
700	150	1.00	47	-167	0.09	0.10	0.42
830	150	0.70	53	-190	0.11	0.12	0.37
1000	150	0.53	60	-218	0.14	0.16	0.33
1200	150	0.44	66	-248	0.18	0.22	0.31
1400	150	0.38	72	-276	0.21	0.27	0.28

Table 1: Values of $g(Q_6)^0_P$ and $g(Q_6)_P$, for constant and momentum dependent penguin coefficient respectively, calculated for different values of Λ and M. The corresponding modified values $g(Q_6)^M_P$obtained when compensating for wrong values obtained for f_π and $<\bar{q}q>$ are also given.

Both for Q_6 and Q_4, our procedure makes the χQM calculation sensitive to m_c, which does not sound unreasonable because the scale m_c is not too far above Λ_χ. However, even if matrix elements of the penguin contribution are sensitive to the scale m_c, we expect that the effect of high momenta $\sim m_c$ should be damped. Especially, we expect the leading behaviour $\sim m_c^2$ in (34) to be damped due to some cut-off Λ. Therefore we have studied the Q_6 amplitude by using proper time regularization. Then the two loop integral can not be solved analytically, but we can perform a numerical calculation for various values of the cut-off Λ. This way of regularization will have an exponential damping of higher momenta. For this calculation we have used (6) with $\alpha_s = \alpha_s(\Lambda)$.

The values found for $g_8^{(1/2)}(Q_6)_P \equiv g(Q_6)_P$ for finite cut-offs Λ calculated within proper time regularization are tabulated in table 1. For comparison, the corresponding values $g_8^{(1/2)}(Q_6)^0_P \equiv g(Q_6)^0_P$ for a constant penguin coefficient (using (5) with $\mu = \Lambda$) are also given. As mentioned above, the integral for $g(Q_6)$ contains the product of a quadratic divergence corresponding to the quark condensate (see (21)) and a logarithmic divergence

for f_π (see (20)). One should note that for various values of Λ and M, the obtained values for f_π and $< \overline{q}q >$ are rather different from the physical values (The gluon condensates in (20) and (21) are not included here). This leads to wrong values for $g(Q_6)$ both for a constant and a momentum dependent penguin coefficient. To compensate for this, we have calculated the modified values $g(Q_6)_P^M$ obtained when multiplying with the physical f_π and $< \overline{q}q >$, and dividing with the respective values $f_\pi(\Lambda, M)$ and $< \overline{q}q > (\Lambda, M)$ in table 1. This procedure should correspond to the necessity of keeping the relation $f_\pi^{(0)} = f$ as mentioned below (25). For the physical quark condensate, we have used $< \overline{q}q > (\mu = \Lambda)$ from formula (26), with $< \overline{q}q > (\mu = 1\mathrm{G}eV) \simeq (-235\mathrm{Me}V)^3$, corresponding to $(m_u + m_d) \simeq 12$ MeV at $\mu = 1$ GeV.

6. Discussion

We have investigated the consequences of taking into account the momentum dependence of the penguin coefficient for $K \to 2\pi$ decays. The calculations are performed within the Chiral Quark Model (χQM). It might seem surprising that the penguin amplitude corresponding to Q_6 gives such a big leading contribution within dimensional regularization. But we will point out that a similar result was obtained[26] for the so called siamese penguin diagram, where we found a result $\sim m_c^2$ in the limit $\Lambda^2 \to \infty$, instead of the "expected" $\sim \Lambda^2$. Unfortunately the dimensional regularization case, which is the only one which can be done analytically for Q_6, is hard to analyze numerically because dimensional regularization does not distuinguish quadratic and logarithmic divergences. One might also worry if dimensional regularization includes too high loop momenta within the χQM. On the other hand, the dimensional regularization result for Q_4 seems to be reasonable.

For finite values of the cut off we have performed a numerical integration. Using proper time regularization, we obtain a $10 - 20\%$ increase of the penguin contribution compared to the case where the penguin coefficient is considered as momentum independent. The reason that we get a slightly bigger value for $g(Q_6)_P$ than $g(Q_6)_P^0$ is the following: The relevant diagrams for the $K \to \pi$ transition to order k^2 are the "eight" diagram (Fig.3a) and the "keyhole" diagram (Fig.3b). For a constant penguin coefficient, it is a partial cancellation between these diagrams (in the order k^2 terms) which makes the small L_5 in (24). For a momentum *dependent* penguin coefficient, the "eight" and the "keyhole" diagrams are influenced in different ways, and $g(Q_6)_P$ is not anymore directly proportional to L_5, but to some "smeared out" and slightly bigger analogue of L_5.

We have not considered the effect of the momentum dependence of the

coefficients $C_{1,2}$ (or equivalently $C_\pm$) of the non-penguin operators $Q_{1,2}\,(Q_\pm)$ in (13). However, these coefficients are not so sensitive to the involved parameters, and we expect even smaller effect of momentum dependence than for penguin-operators. Furthermore, we have not taken advantage of the renormalization group analysis involving the penguin coefficient C_6. This is because the individual quark momenta can hardly be distinguished, owing to the mixing of operators.

Our analysis offers no solution to the $\Delta I = 1/2$ puzzle for $K \to 2\pi$ decays. Moreover, the 10-20% increase of the penguin contribution seen in table 1 is moderate compared to the increase in the penguin coefficient C_6 owing to two loop effects [19]. The penguin contribution to $K \to 2\pi$ is very sensitive to the involved parameters as the quark condensate, the constituent quark mass M, and the coefficient C_6 through Λ_{QCD} and the matching scale μ. In conclusion, other contributions than $g_8^{(1/2)}(Q_6)$ must be important for the $\Delta I = 1/2$ enhancement [25].

7. Appendix

We will here give some details from the calculation of I_χ in (31). We expand the propagators up to second order in the external virtual meson momentum k. Then the propagators $[r^2 - M^2]^{-1}$ and $[(r-p)^2 - M^2]^{-1}$ occur in some powers. We need one Feynman parameter (x) to perform the integral. We use the general formula

$$\int \frac{d^D r}{(2\pi)^D}\,[r^2 - A]^{-n} \;=\; iJ_n(D)[-A]^{(D/2-n)}\;, \tag{39}$$

where

$$J_n(D) \;=\; \frac{(-\pi)^{D/2}}{(2\pi)^D}\,\frac{\Gamma(n - D/2)}{\Gamma(n)}\;. \tag{40}$$

Note that $J_{n+1}(D) = J_n(D)(n - D/2)/n$. Dropping the constant term $\sim k^0$, we obtain

$$\begin{aligned} I_\chi \;=\; & iJ_3(D)k^2 \int_0^1 dx\,\Big\{ S_c(3)\,\Big[\frac{6}{D}(1 - x(1-x)) - 2 \\ & - (\frac{6-D}{D})(1 - 2x(1-x))\Big] + (\frac{6-D}{D})x(1-x)\,M^2\,S_c(4)\Big\} \\ & - (c \;\to\; u)\;, \end{aligned} \tag{41}$$

where the quantity $S_q(n)$ depends on the current quark masses for $q = c, u$:

$$S_q(n) \;=\; \int \frac{d^D p}{(2\pi)^D}\,[C_P(p^2)]_q\,[-A]^{(D/2-n)}\;, \tag{42}$$

where $[C_P(p^2)] = [C_P(p^2)]_u - [C_P(p^2)]_c$, and $A = M^2 - x\,(1 - x)\,p^2$. To find $S_q(n)$, we have to use the dimensional regularization version of $[C_P(p^2)]$ in (7), and we need the generalized Feynman parametrization

$$\frac{1}{(-A)^l(-D_q)^\epsilon} \;=\; \frac{1}{B(l,\epsilon)} \int_0^1 dy\, \frac{(1-y)^{l-1} y^{\epsilon-1}}{N_q^{l+\epsilon}} \tag{43}$$

where $l = n - D/2$ and $N_q = (1-y)(-A) + y(-D_q) = h(p^2 - \overline{m_q}^2)$. Here, $h = yt(1-t) + (1-y)x(1-x)$ and $\overline{m_q}^2 = (y m_q^2 + (1-y)M^2)/h$. The quantity B is given by

$$B(a,b) \;=\; \int_0^1 dx\, x^{a-1}\,(1-x)^{b-1} \;=\; \frac{\Gamma(a)\cdot\Gamma(b)}{\Gamma(a+b)} \tag{44}$$

Using again (39), we obtain

$$S_q(n) \;=\; iJ_{l+\epsilon}(D)\frac{\Gamma(\epsilon)}{B(l,\epsilon)} \int_0^1 dt\, 6t(1-t) \int_0^1 dy\, \frac{(1-y)^{l-1} y^{\epsilon-1}}{h^{l+\epsilon}}\,(-\overline{m_q}^2)^u\,, \tag{45}$$

where $u = D/2 - (l+\epsilon)$. Now, for the charm quark case $q = c$, we use the approximation $\overline{m_c}^2 = y m_c^2/h$. Then we obtain

$$S_c(n) \;=\; iJ_{l+\epsilon}(D)\frac{\Gamma(\epsilon)}{B(l,\epsilon)}(-m_c^2)^u \int_0^1 dt\, 6t(1-t)\, I(x,t) \tag{46}$$

Then the clue is to recognize the integral

$$I(x,t) \;=\; \int_0^1 dy\, \frac{(1-y)^{l-1} y^{\epsilon+u-1}}{h^{u+l+\epsilon}} \;=\; \frac{B(l,u+\epsilon)}{[x(1-x)]^l[t(1-t)]^{u+\epsilon}} \tag{47}$$

which gives

$$S_c(n) \;=\; \overline{S_c(n)}\,[x(1-x)]^{-l} \tag{48}$$

with

$$\overline{S_c(n)} \;=\; iJ_{l+\epsilon}(D)\frac{\Gamma(\epsilon)}{B(l,\epsilon)}(-m_c^2)^u\, B(l,u+\epsilon)\, 6\, B(2-u-\epsilon, 2-u-\epsilon) \tag{49}$$

Then, to leading order in m_c^2, we find

$$I_\chi \;=\; -iJ_3(D)k^2\overline{S_c(3)}\,B(D/2-2, D/2-2) \tag{50}$$

Here the factor $B(D/2-2, D/2-2)$ corresponds to the logarithmic divergence connected to f_π, and $J_{l+\epsilon}(D)$ in $S_c(3)$ to the quadratic divergence connected to the quark condensate.

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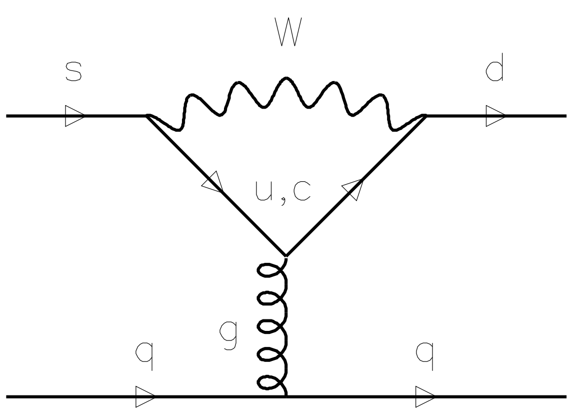

Figure 1: The penguin loop diagram. At the lower line $q = u, d, s$.

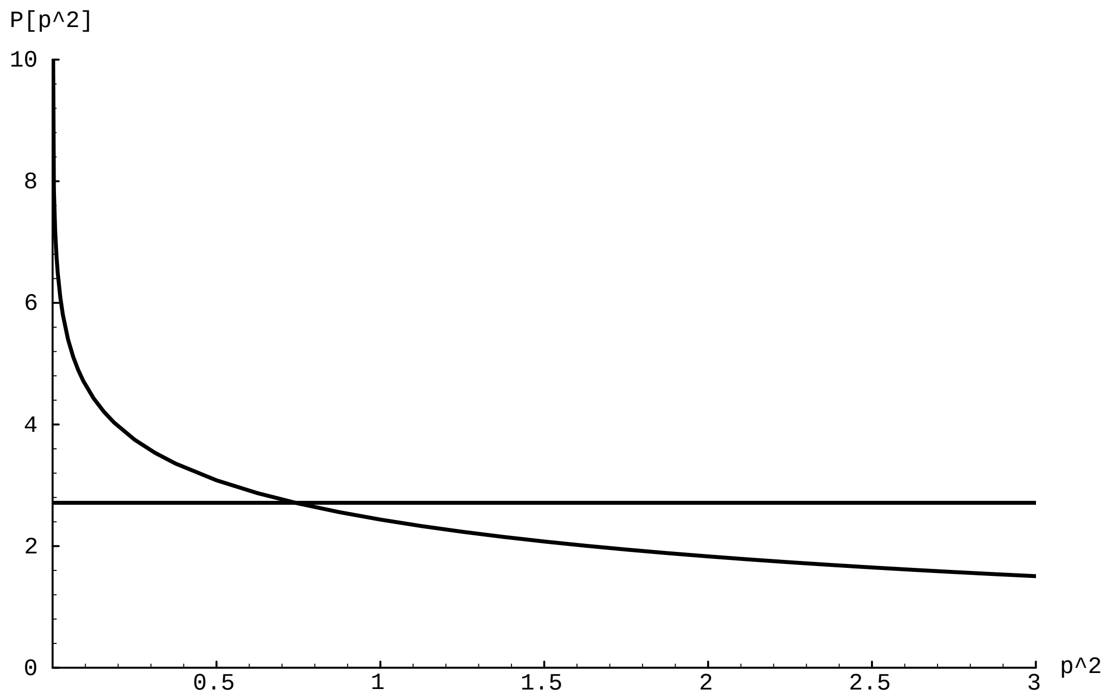

Figure 2: Variation of the penguin coefficient $C_P = -\frac{\alpha_s}{3\pi}\, P(p^2)$ in (6) with squared Euclidean momentum p^2 (-in units GeV^2). The horisontal line represents the value in eq. (5) for $\mu = 0.83$ GeV.

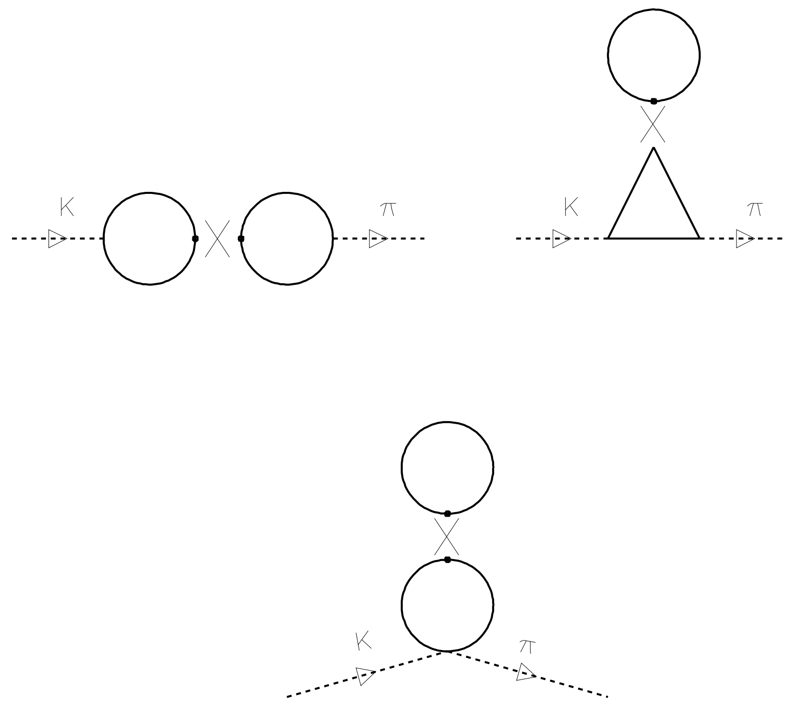

Figure 3: Diagrams for the penguin operator Q_6 contribution to $K \to \pi$. The solid lines represent quarks.Diagram (a), upper left, is the "eight" diagram, while (b) is the "keyhole" diagram. Diagram (c) serves to cancel the momentum independent part of (a) and (b).